

Calc II - Review for exam II

The first exam will be this Friday, October 23. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Suppose it requires a force of 20N to hold a spring 0.1 meters past its natural position. How much work is required to get it there?
2. A 100 meter rope with linear density 1kg/m dangles over the edge of a cliff. How much work is required to pull the rope up to the top of the cliff?
3. Evaluate the following improper integrals, or state why they diverge.

(a) $\int_0^{\infty} e^{-2x} dx$

(b) $\int_1^{\infty} \frac{\ln(x)}{x^2} dx$

(c) $\int_1^{\infty} \frac{1}{x^3} dx$

4. Write one or two complete sentences carefully explaining why $\int_2^{\infty} \frac{x^{41}}{x^{42} - 1} dx$ diverges.
5. Write down a complete sentence proving that the improper integral $\int_1^{\infty} e^{-x^2} dx$ converges.
Note: You may assume that $\int_1^{\infty} e^{-x} dx$ converges.
6. Use u -substitution to express the following normal integral as a standard normal integral:

$$\frac{1}{2\sqrt{2\pi}} \int_0^4 e^{-(x-3)^2/8} dx$$

7. Suppose that an unfair coin comes up heads $3/5$ of the time. Each time it comes up heads, we write down a one and each time it comes up tails, we write down a zero.
 - (a) Compute the mean and standard deviation associated with one such flip.
 - (b) Use a normal integral to estimate the probability that we get more than 125 heads in 200 flips.
8. I have a six sided die with two sides labeled 1, two sides labeled 2, and two sides labeled 3; thus, it generates a one, a two or a three with equal probability $1/3$. Now suppose I roll that die 150 times and add the resulting numbers.
 - (a) Write down a sum that shows the mean of one roll is $\mu = 2$. What is the mean associated with 150 rolls?
 - (b) Write down a sum that shows the variance of one roll is $\sigma^2 = 2/3$. What is the variance associated with 150 rolls?
 - (c) Write down a normal integral representing the probability that the sum total of my 150 rolls is more than 290 but not more than 310.

9. Suppose we wish to estimate

$$\int_0^3 \sqrt{x^3 + 1} dx$$

with a midpoint sum and we'd like our result to be within 0.0001 of the actual value.

- Find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.
- Write down the resulting sum using summation notation.

10. Suppose we use Simpson's rule with $n = 6$ to estimate

$$\int_0^3 \sqrt{x^3 + 1} dx$$

- Write down the resulting sum.
- What is the error estimate.

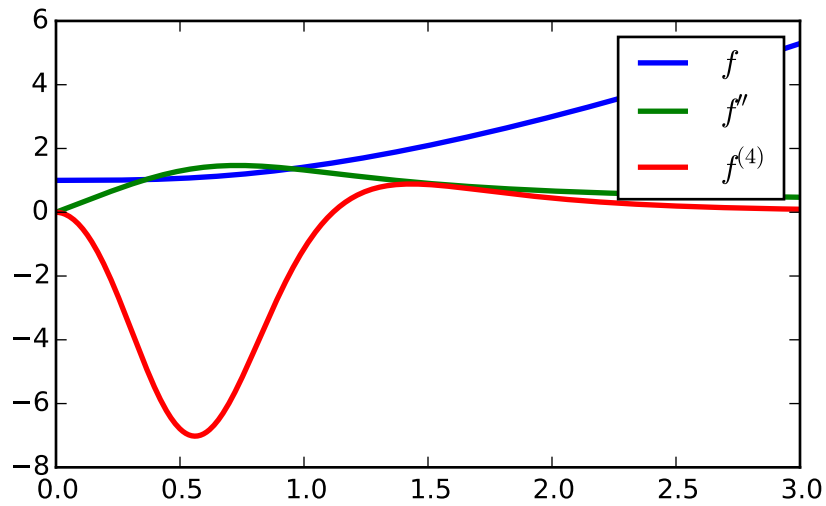


Figure 1: The graphs of $f(x) = \sqrt{x^3 + 1}$, $f''(x)$, and $f^{(4)}(x)$