Calc II - Problems off of past exams +

Our final exam will be comprehensive. Here are *most* of the problems off of past exams to help focus your studies - as well as some problems like our last problem sheet.

Exam 1

1. Evaluate the following integrals using the technique indicated.
   (a) $\int_{-1}^{1} x^3 \sqrt{x^4+1}\, dx$ - using u-subs
   (b) $\int x \sin(2x) dx$ - by parts
   (c) $\int \frac{x^2+x}{(x-1)(x^2+1)} dx$ - using partial fractions
3. Suppose we wish to estimate
$$\int_0^2 \sin(x^2) dx$$
with a midpoint sum and we'd like our result to be within 0.0001 of the actual value.
   (a) Find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.
   (b) Write down the resulting sum using summation notation.

   Note that the graph of $f(x) = \sin(x^2)$ together with its second derivative is show in figure 1.
4. Evaluate the following integrals using any technique that you see fit.
   (a) $\int_0^3 \sqrt{9-x^2}\, dx$
   (b) $\int_0^3 x^2 \sqrt{9-x^2}\, dx$
   (c) $\int_0^\pi \sin^3(x) \cos^2(x)\, dx$
   (d) $\int \sqrt{1+x^2} dx$

Exam 2

1. (ish) Suppose we spin the region under the graph of $f(x) = 1/x^3$, to the right of the line $x = 1$, and over the x-axis around the x-axis. What is the volume of the resulting solid?
5. Use u-substitution to express the following normal integral as a standard normal integral:
$$\frac{1}{3\sqrt{2\pi}} \int_0^5 e^{-(x-2)^2/18}\, dx$$
6. Let
$$p(x) = \begin{cases} \frac{3}{(x+1)^4} & x \geq 0 \\ 0 & x < 0. \end{cases}$$

(a) Show that p is a good probability distribution.

(b) Compute the mean of p.

(c) If X is a random variable with distribution p, compute $P(1 \leq X \leq 3)$.

7. I have a ten sided die with two sides labeled 1, six sides labeled 2, and two sides labeled 3. Now suppose I roll that die 500 times and add the resulting numbers.

   (a) What is the mean μ associated with one roll of this die. What is the mean associated with 500 rolls?

   (b) What is the variance σ^2 associated with one roll of this die. What is the variance associated with 500 rolls?

   (c) Write down a normal integral representing the probability that the sum total of my 500 rolls is more than 240 but not more than 280.

8. The object shown in the figure 1 below has horizontal slices that are all rectangles twice as long as they are wide. The base has larger side length four, the top has larger side length two, and the height of the object is three. What is the volume of the object?

Exam 3

1. Evaluate $\displaystyle\sum_{n=2}^{\infty} 2\frac{(-4)^n}{3^{2n}}$.

2. Suppose we'd like to approximate
$$\sum_{n=1}^{\infty} \frac{1}{n^4+1}$$
by truncating the sum to obtain a finite sum of the form
$$\sum_{n=1}^{N} \frac{1}{n^4+1}.$$
How large does N have to be to ensure that our approximation is within 0.001 of the actual value?

3. Write down a couple complete sentences using the alternating series test to show that
$$\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{\sqrt{n}}$$
converges conditionally.

5. Classify the following series as absolutely convergent, conditionally convergent, or divergent. Note: You need not justify your assertion.

   (a) $\displaystyle\sum (-1)^n \frac{\sqrt{n}}{\sqrt{n^2+1}}$

   (b) $\displaystyle\sum (-1)^n \frac{\sqrt{n}}{\sqrt{n^3+1}}$

   (c) $\displaystyle\sum (-1)^n \frac{\sqrt{n}}{\sqrt{n^5+1}}$

6. Find the domain of convergence of the power series $\displaystyle\sum (-1)^n \frac{3^{n+1}}{n^2} x^n$.

7. Starting with the geometric series formula, find a power series for $1/(1+x^2)$, then integrate to find a series representation of

$$\int \frac{1}{1+x^2}\,dx = \arctan(x).$$

A couple more problems

1. Find the cubic approximation of $f(x) = \sqrt{x-1}$ at $x = 5$.
2. Let $f(x) = \cos(2x)$. Use Taylor's formula to find the series expansion of f about the point $c = \pi/2$.
3. What function does $\sum_{n=1}^{\infty} n^2 x^n$ represent?

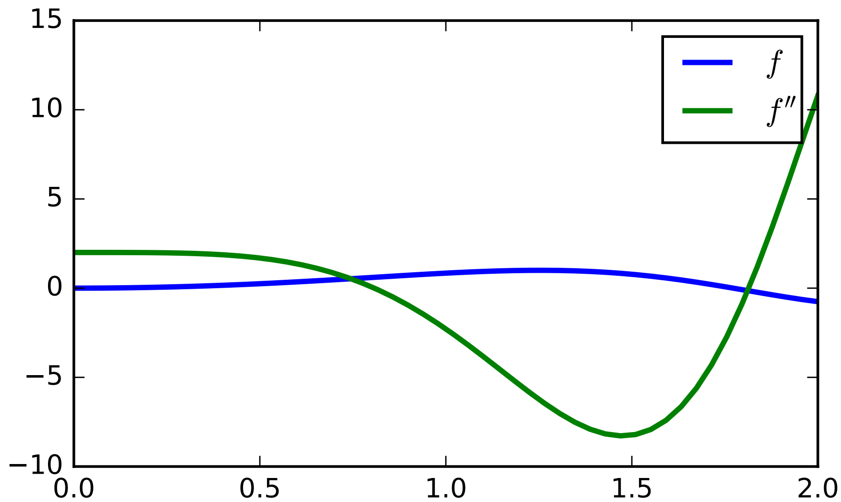

Figure 1: The graph of $f(x) = \sin(x^2)$ together with its second derivative

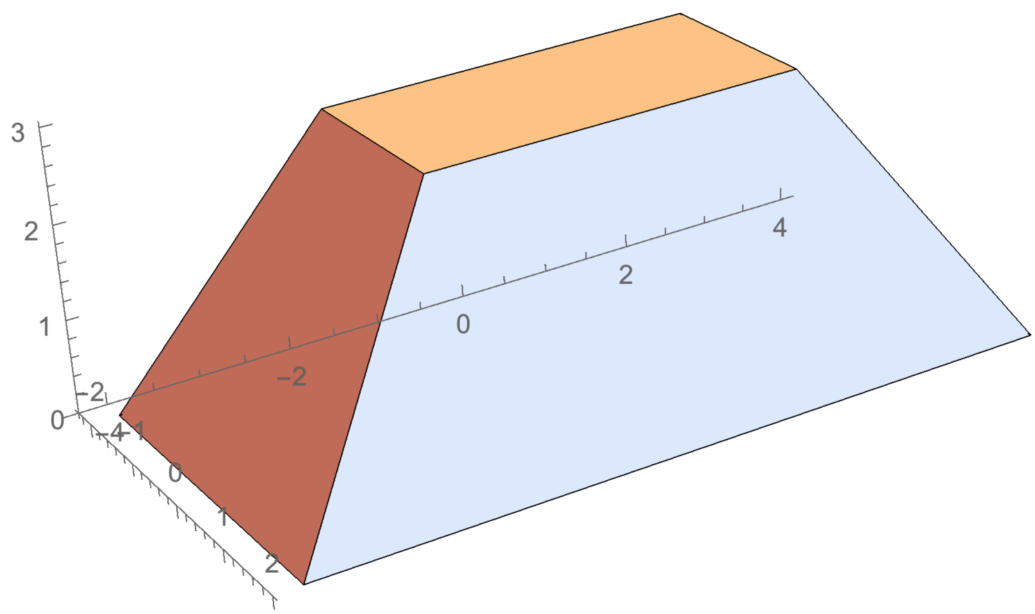

Figure 2: The 3D object for the last problem.