Calc I - Past Exam Problems

Our final exam is this coming Friday, April 29 at 11:30 in our usual class room. In addition to the u-substitution problems listed in our text, in your WebWork, and on our last day's problem sheet, you should study the problems off of past exams. For your convenience and focus I've listed most of those here.

Exam I

3. Find the derivatives of the following functions, *using the definition of the derivative.*
   (a) $f(x) = x^2 - 3x + 2$
   (b) $f(x) = x^4$
4. Find the derivatives of the following functions, using any technique you see fit.
   (a) $f(x) = x^2 - 3x + 2$
   (b) $f(x) = 1/\sqrt{x}$
   (c) $f(x) = \dfrac{1 - x - x^2}{x}$
   (d) $f(x) = e^x(1 - x)$
5. Let $f(x) = x^2 - 3x + 2$. Find an equation of the line that is tangent to the graph of f at the point where $x = 2$.
7. The complete graph of a function f is shown in figure 1 below. Sketch a graph of f' on the axes below the graph of f.

Exam II

3. Write the equation $y = x^{-x^2}$ as $\ln(y) = \ln(x^{-x^2})$ and differentiate both sides implicitly with respect to x to find y' in terms of x.
4. Use the differentiation rules to find f' for each of the following functions.
   (a) $f(x) = \sin(x) + \cos(x) + \ln(x) + e^x$
   (b) $f(x) = xe^{-x^2}$
   (c) $f(x) = \arcsin(-x^2)$
5. Find an equation of the line tangent to the curve $x^4y + 3xy^3 = -2$ at the point $(-1, 1)$.

6. Find the absolute maximum and minimum of $f(x) = \frac{x}{2} + \frac{1}{x}$ over the interval $[1, 4]$.

Exam III

1. The graph of $f(x) = 2xe^{-x^2} - x$ is shown in figure 2.
   (a) Write down an equation that the critical points of f must satisfy.
   (b) Suppose we wanted to find the smallest, positive critical point of f using Newton's method. Write down the corresponding Newton's method iteration function and a reasonable initial guess to start the iteration.
   (c) Find the exact values of the inflection points of f and indicate their positions in the graph.
2. Let $f(x) = (x + 1)(x - 1)^3$
   (a) Find all the critical points of f.
   (b) Find all the inflection points of f.
   (c) Sketch a rough graph of f.
3. The bottom of a 12 foot long ladder slides away from a wall at 2 feet per second. How fast is the top of the ladder moving toward from the floor when the bottom is 11 feet away from the wall?
4. Find the most general anti-derivatives of the following functions.
   (a) $f(x) = 5x^4 - 2x^3 + x^2 - 4x$
   (b) $f(x) = e^x + \sin(x) - \cos(x)$
5. Suppose an object starts at height $y_0 = 10$, is thrown upward with initial velocity $v_0 = 8$ and moves with constant acceleration $a(t) = -32$.
   (a) Find a height function $y(t)$ for the object.
   (b) How fast does the object hit the ground?
6. Suppose I set up a rectangular corral with inner partitions, as shown in figure 3. The material for the exterior portion costs three times as much as the material for the interior walls and the total area must be 1000 square feet. What are the dimensions of the cheapest such corral?
7. The graph of a function f is shown in figure 4; it consists of a semi-circle and two straight line segments. Compute

$$\int_{-1}^{3} f(x)dx.$$

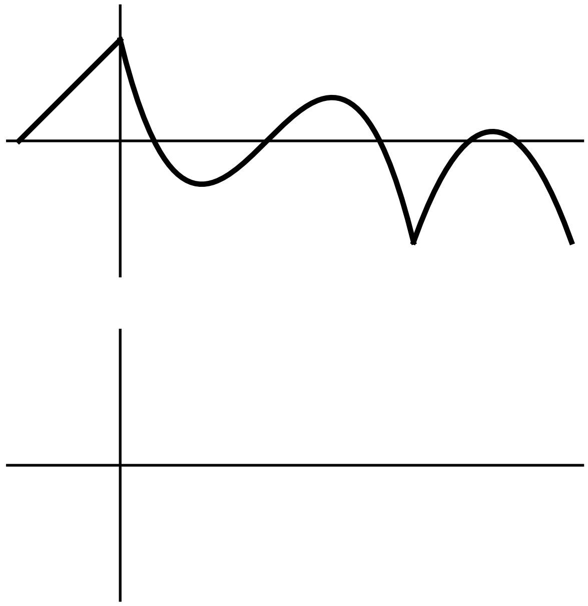

Figure 1: The graph of a function with a spare set of axes

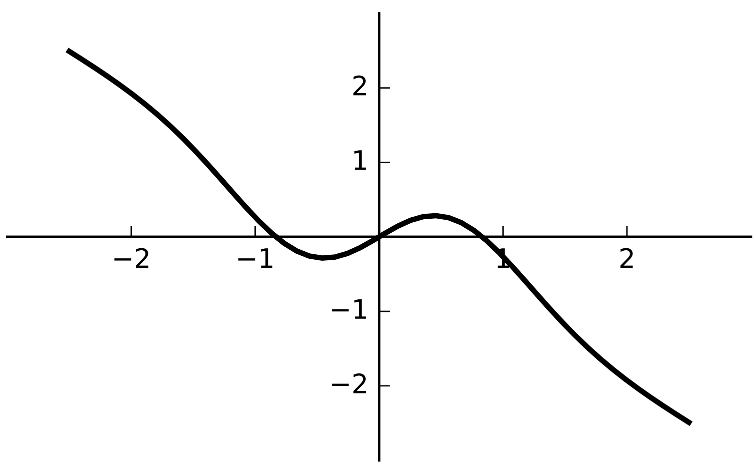

Figure 2: The graph of $f(x) = 2xe^{-x^2} - x$

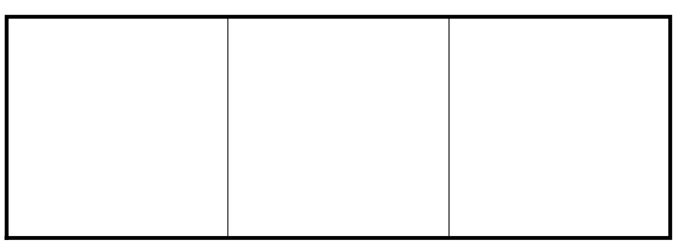

Figure 3: A corral

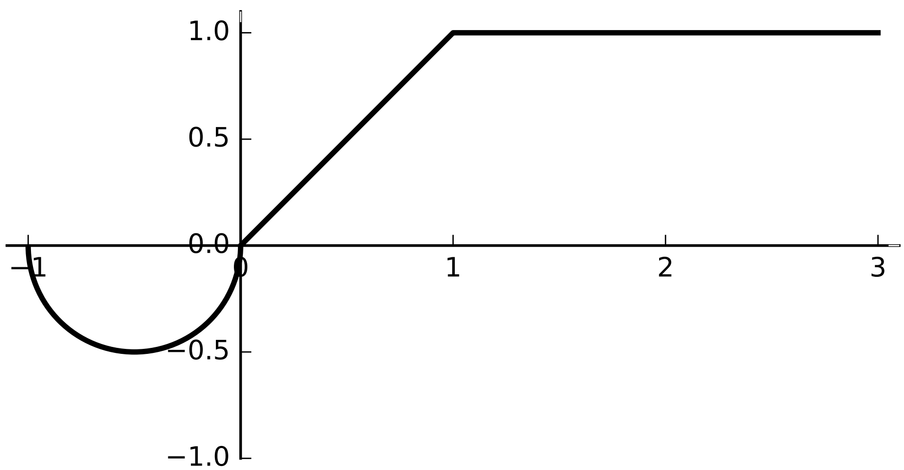

Figure 4: The plot of $f(x)$ for problem 8