Problems - HW 6

Thursday, October 3

1. Suppose that V is a vector space of dimension 7, that W is a vector space of dimension 4, and that $T : V \to W$ is a linear transformation. Which of the following is *not* necessarily true?

   (a) If $T(v) = 3w$, then $T(2v) = 6w$.

   (b) There are independent vectors $v_1, v_2 \in V$ such that $T(v_1) = T(v_2) = 0$.

   (c) $T(v_1 - v_2) = T(v_1) - T(v_2)$.

   (d) For every $w \in W$, there is a $v \in V$ such that $T(v) = w$.

   (e) There are independent vectors $v_1, v_2 \in V$ such that $T(v_1) = T(v_2)$.

2. If V and W are subspaces of $\mathbb{R}^8$ with $\dim(V) = 5$ and $\dim(W) = 6$, then which of the following must be true.

   (a) $\dim(V \cap W) = 1$

   (b) $\dim(V \cap W) = 5$

   (c) $3 \leq \dim(V \cap W) \leq 5$

   (d) $7 \leq \dim(V + W) \leq 10$

   (e) $\dim(V + W) = 11$

3. Which of the following is *not* a subspace of $\mathbb{R}^3$?

   (a) $\{(x, y, z) \in \mathbb{R}^3 : x - 3y + 2z = 0\}$

   (b) $\{(x, y, z) \in \mathbb{R}^3 : x^2 - y^2 + z^2 = 0\}$

   (c) $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 0\}$

   (d) The y-axis

   (e) The xz-plane