Problems - HW 5

Thursday, September 26

1. For which values of c does the following system have no solution?

$$\begin{aligned} x - y + z &= 3 \\ x + y + z &= 1 \\ x - y + (c^2 - 8)z &= c \end{aligned}$$

2. As is well known, three non-collinear points in the plane determine a unique circle. There's a very groovy way to find the equation of that circle using a determinant. Let the points be (x_1, y_1), (x_2, y_2), and (x_3, y_3). An equation for the circle is then

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0.$$

   (a) Apply this technique to find an equation of the circle containing the points $(1, 1)$, $(2, 3)$, and $(3, 4)$.

   (b) What happens when the three points are collinear? Illustrate with an example.

   (c) Generalize this technique to find the equation of a sphere containing four points in space. Apply your technique to find an equation of the sphere containing the points $(1, 1, 1)$, $(2, 3, 4)$, $(3, 4, 5)$, and $(6, -6, 6)$.