Problems - HW 4

Thursday, September 19

1. Describe the geometric action of the following matrices using terms such as reflection, rotation, dilation, and skew.

   (a) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

   (b) $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

   (c) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

   (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2. Let M denote the matrix

$$M = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$$

   (a) M represents a reflection about what line?
   Your answer should be in terms of θ.

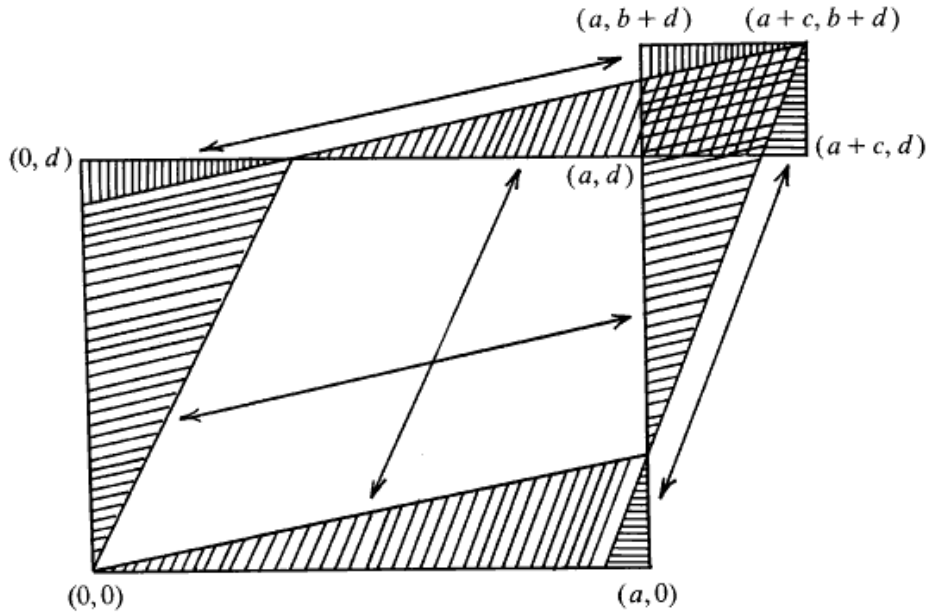
   (b) Use this formulation to find a reflection about the line $y = \sqrt{3}x$.

3. Let A be the 2×2 matrix

$$A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}.$$

   Use the figure on the reverse to explain why the area of the parallelogram spanned by the column vectors of A is $ad - bc$.
   Acknowledgement: The figure comes from math.stackexchange. Evidently, it originally appeared in the 1985 Math Mag.

Proof without words:
A 2×2 determinant is the area of a parallelogram



$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc =$$