Quiz 1 Review problems

1. Write down a careful definition of each of the following.

   (a) The set of complex numbers
   (b) The fundamental theorem of algebra
   (c) The absolute value of a complex number
   (d) The argument of a complex number
   (e) The triangle inequality
   (f) Interior point
   (g) Boundary point
   (h) Open set
   (i) Closed set
   (j) Path
   (k) Limit
   (l) Continuity

2. Show that complex multiplication is a commutative operation.

3. Let $a, b \in \mathbb{C}$ and $f(z) = az + b$. Show that $\lim_{z \to z_0} f(z) = f(z)$.

4. Show that $\mathbb{R}$ is a closed subset of $\mathbb{C}$.

5. Show that $f(z) = \overline{z}$ is nowhere differentiable.

6. Show that $|z| = 1$ iff $z = \overline{z}$.

7. Express $(\sqrt{3} + i)^{100}$ in the form $a + bi$.

8. Describe the following sets and classify them as open, closed, or neither.

   (a) $\{z : 1 < |z + 1 - 2i| \leq 2\}$
   (b) $\{z : \text{Re}(z) > 0\}$
   (c) $e^{\pi i/4}\{z : \text{Re}(z) > 0\}$
   (d) $\{z : |z - (1 + i)| = |z + (1 + i)|\}$