

Calc II - Problems off of past exams

The final exam for this class is scheduled for next Wednesday, December 2 at 11:30 in our usual class room. You can treat the past exams as review sheets with a focus on the problems listed here. It wouldn't hurt to look at the past review sheet problems that are like these, as well.

Exam I

1. Evaluate the following integrals using the technique indicated.

(a) $\int x\sqrt{x^2+1} dx$ - u -subs

(b) $\int x \ln(x) dx$ - by parts

2. Suppose we know that

$$\int_{-1}^1 f(x) dx = 8.$$

Which of the following integrals can we compute and what is the value?

(a) $\int_0^4 f\left(\frac{x-2}{2}\right) dx$

(b) $\int_0^1 f(x) dx$

4. (a) Use integration by parts to derive a reduction formula for the integral $\int x^n e^x dx$.

(b) Use your reduction formula to evaluate $\int x^4 e^x dx$.

7. Evaluate the following integrals using any technique that you see fit.

(a) $\int x^3 \sqrt{x^2+1} dx$

(b) $\int_0^\pi \cos^4(x) dx$

(c) $\int_0^1 x^5 \sqrt{1-x^2} dx$

Exam II

1. Suppose it requires a force of 10N to hold a spring 0.2 meters past it's natural position. How much work is required to get it there?
2. Evaluate the following improper integrals, or state why they diverge.

(a) $\int_1^\infty \frac{1}{x^4} dx$

(b) $\int_0^{\infty} xe^{-x^2} dx$

3. Write one or two complete sentences carefully explaining why $\int_1^{\infty} \frac{\ln(x)}{x} dx$ diverges.
4. Use u -substitution to express the following normal integral as a standard normal integral:

$$\frac{1}{3\sqrt{2\pi}} \int_0^5 e^{-(x-2)^2/18} dx$$

5. I have a six sided die with three sides labeled 1, two sides labeled 2, and one sides labeled 3; thus, it generates a one, a two or a three with unequal probabilities. Now suppose I roll that die 99 times and add the resulting numbers.
- (a) Write down a sum that shows the mean of one roll is $\mu = 5/3$. What is the mean associated with 99 rolls?
- (b) Write down a sum that shows the variance of one roll is $\sigma^2 = 5/9$. What is the variance associated with 99 rolls?
- (c) Write down a normal integral representing the probability that the sum total of my 99 rolls is more than 105.
6. Suppose we wish to estimate

$$\int_0^2 e^{-x^2} dx$$

with a trapezoidal sum and we'd like our result to be within 0.00001 of the actual value.

- (a) Find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.
- (b) Write down the resulting sum using summation notation.

Exam III

1. Suppose we spin the region under the graph of $f(x) = 5x(1-x)$ and over the x -axis around the x -axis. What is the volume of the resulting solid?
2. My old tent is three feet high and is held up by two pieces of tent cord so that the base is a square. What is the volume of that tent?
3. Write down an integral representing the arc length of the graph of $y = x^3$ over the interval $[0, 3]$. You do *not* have to evaluate the integral.
4. Write down a complete sentence or two referring to the comparison test and the appropriate p -series to show that

$$\sum_{n=2}^{\infty} \frac{n+1}{n^2-1}$$

diverges.

5. Evaluate the sum $\sum_{n=2}^{\infty} (-1)^n \frac{3^{n+2}}{5^n}$.
6. Use the geometric series formula to express $8.\overline{03}$ as a fraction.
7. Let $f(x) = \sqrt{x}$. Find the quadratic approximation of f centered at $x_0 = 1$.
8. Use a power series expansion to evaluate the integral $\int e^{-x^2} dx$.